Reference system and non-rotating origin: the NRO for the rest of us
Olivier de Viron, Véronique Dehant, and Nicole Capitaine

Abstract
The concept of Non Rotating Origin has been introduced in the method used to express in the Celestial reference frame a quantity that was expressed in the Terrestrial reference frame. The main changes lie in the new definition of the intermediate frame. This paper proposes and comments 3D computer animations of these changes of reference frame developed for non-experts in the field.

Introduction
The purpose of this paper is to introduce the recent changes in the representation of the Earth’s rotation in space to the non expert in the field of Earth rotation. We have developed a set of 3D computer animations in order to illustrate and explain those changes.

In order to describe and study any phenomenon at the Earth surface or in space, it is necessary to pinpoint its location with respect to a conventional reference system. In particular, when a phenomenon occurs at the Earth surface, it is efficient to refer it to a system moving with the Earth, which mimics its rotation. Moreover, such a system has to be the object of an international consent and adopted internationally, in order to be able to characterize phenomena at global scale, and for instance, to compute distance at the continental scale. With no international agreement on a global reference system, the same location would have a different position depending on the system to which it is referred; some decades ago, there was for instance a two-meter difference between an altitude referred to the Belgian reference system and the same point in the French reference system. For astronomy and geodesy in particular, an international Earth-based reference system, the International Terrestrial Reference System (ITRS) has been defined several decades ago (as explained in Altamimi et al., 2002) by the International Astronomical Union (IAU) and the International Association for Geophysics and Geodesy (IUGG). The coordinate system of the ITRS is materialized by a set of station position time-series defining the realization of the ITRS, called the International Terrestrial Reference Frame (ITRF). This reference frame is represented in the 3D-animation called http://www.astro.oma.be/D1/DIDAC/NROtext/TRF.wmv

As a raw approximation, the \( z \)-axis of the reference system is close to the mean Earth rotation axis or equivalently the mean figure axis; the \( x \)-axis is close to the intersection between the equator and the Earth prime meridian (Greenwich) and the \( y \)-axis complete the right-handed reference frame. To remain consistent with time, the definition of the ITRS has a constraint of no global rotation (including perturbations from tectonic plate motion). In 1985, it was chosen to use, for the reference frame definition, a set of well observed station positions and velocities at the Earth surface, supplemented with a plate tectonics model. Of course, due to plate tectonics and ground deformations, it is necessary to continuously monitor the station positions (and velocities) with time, to ensure a stable realization of the reference frame. The latest realization of the ITRS has been computed in 2005.

On the other hand, when observing objects in the sky, it is necessary to refer their direction with to a celestial reference system, fixed in space and not rotating with the Earth. In such a system, a fixed object in space, if such an object exists, would have no
motion. The IAU has defined the International Celestial Reference System (ICRS). In order to be consistent with the previous celestial system defined from a catalogue of stars and conventionally linked to the mean ecliptic and the mean equator of a fixed epoch, the ICRS Z-axis was chosen to be close to the mean Earth rotation axis or figure axis, at noon, on January 1st, 2000 (J2000), and the X-axis close to the mean equinox (vernal point) for the epoch J2000, the Y-axis completing a right-handed reference system (see the 3D-animation called http://www.astro.oma.be/D1/DIDAC/NROtext/CRF.wmv

The realization of the ICRS, the International Celestial Reference Frame (ICRF), consists in a list of quasars and their coordinates (Ma et al., 1998). The quasars are galaxy cores, very far away, emitting time-variable radio-signals. Due to this large distance, their angular motions, as observed from the Earth, are very small. Due to their internal dynamics, the positions of the emitting part of quasars may change with time, and those positions have to be monitored to ensure the stability of the reference system. The so-called “defining sources” are in principle the most stable sources of the ICRF.

It makes more sense to define the position of stars in the celestial reference frame, whereas the observing stations are on the ground. Consequently, a reference frame transformation, making it possible to convert positions expressed in the ICRF to positions expressed in the ITRF and back, is mandatory for astronomic and geodetic studies. In particular, the conversion from the terrestrial to the celestial reference frame requires to precisely determine the Earth rotation (and orientation).

The rotation of the Earth is a complex motion as, in addition to variations of the angular velocity, the orientation of the rotation axis changes in space and the orientation of the Earth changes along its rotation axis; the link between ITRF and ICRF is thus complex, and cannot be precisely represented by a regular rotation only.

In this paper, we describe how it is possible to convert positions referred in the terrestrial (resp. celestial) reference frame to positions expressed in the celestial (resp. terrestrial) reference frame. This conversion has been changed with the new definitions of polar motion, precession and nutation, and Earth rotation angle. We describe how the concept of non-rotating origin (NRO) enters into the game (Guinot, 1979). In Section 1, we describe the kinematics of the Earth rotation, and we give the definitions of polar motion, length-of-day, nutation, and precession. Section 2 is devoted to the change of reference frames, using the classical polar motion, length-of-day, precession-nutation expressions (classical approach) and using expressions based on the concept of non rotating origins (new approach). Some additional issues related to the creation of offsets in the present definitions of pole positions are discussed in Section 3. Conclusions are given in Section 4.

1. The Earth rotation, a complex motion
For a long period of time, the Earth rotation has been used to define the time, one day being the time separating two successive crossing of the meridian by the Sun (solar day) or by a given star (sidereal day). Changes in the Earth rotation were already observed by Hypparcos from Nicea, around 130 BC, who found evidences for the variation of the Earth rotation axis position in space, by comparing his observations of the positions in the sky of well observed stars with respect to the positions of the same stars observed about two centuries earlier by Babylonian astronomers. This observed motion is known as the precession of the equinox, and was explained by Newton. In 1748, Bradley
observed a periodic departure (period $T \approx 18.6$ years) from the precession motion, called nutation. With the improvement of the observation techniques, irregularities in the orientation of the Earth around its rotation axis were observed around 1891 by Chandler; this motion is known as polar motion. Using improved clocks, it was even possible to detect variation in the Earth rotation rate as pointed out by Jackson in 1929. Nevertheless, the Earth rotation was used to define the time and to realize the scientific timescale until the use of Ephemeris Time based on the Earth and Moon orbital dynamics (1956) and then the use of an Atomic Time scale based on atomic clocks, in 1967. Using atomic clocks, as well as astronomic observations and, later, space geodetic techniques, it then became possible to observe, to monitor, and to study the Earth rotation. Mainly three techniques are presently used: (1) the Very Long Baseline Interferometry (VLBI), where the relative orientations of the Earth with respect to quasars are retrieved through the comparison of the arrival time of radio signal emitted by these quasars at stations on the Earth surface, (2) tracking of artificial satellites (such as those from the Global Position System (GPS) or from Satellite Laser Ranging (SLR)) from a station network and accounting for their orbital dynamics, or equivalently from the radio links between gauges and satellites orbiting around the Earth, such as the DORIS system (Doppler Orbitography and Radiopositioning Integrated by Satellite) and (3) Lunar Laser Ranging, considering precisely known ephemerides for the Moon. The orientation of the Earth with respect to the Earth center of mass is obtained by solving the global rotation of the observation network at the Earth surface. The irregularities in the rotational motion of the Earth are presented in the 3D-animation called [http://www.astro.oma.be/D1/DIDAC/NROtext/EarthRotation.wmv](http://www.astro.oma.be/D1/DIDAC/NROtext/EarthRotation.wmv) They were classically divided into three parts: (1) the polar motion, representing the wobble of the Earth with respect to its mean rotation axis, measured in the ITRF, (2) the Earth rotation rate fluctuations, associated with variation of the Length of day (LOD), and (3) the precession-nutation, which is the motion of the Earth mean rotation axis in space (adopted model is MHB2000, Mathews et al., 2002), and consequently measured in the ICRF (see IAU or IUGG resolutions). This division was justified by practical reasons: each of the motions, in the relevant reference frame, was mostly low frequency. Polar motion contains mainly seasonal components and the Chandler Wobble component at a period of $\sim 434$ days in a frame tied to the Earth; the precession-nutation motion contains several slow periodic components in space.

2. Change of reference frame, the classical way and the new way
In view of the main long-term components of polar motion in the terrestrial frame and of precession and nutation in the celestial frame explained in the previous section, a Celestial Ephemeris pole (CEP) was defined (Seidelmann, 1982), which had long period polar motion components in the Earth reference frame, and a precession-nutation motion that has long period components in space. The length-of-day variations are mainly slow (seasonal, decadal, trend) in the celestial reference frame. The typical sizes of those irregularities are of the order of the millisecond for the LOD variations, several tens of meters for the polar motion (as seen in a plane perpendicular to the $z$-axis at the Earth surface), and several hundreds of meters per year and several hundreds of meters for the precession and nutation respectively (as seen in space in a plane tangential to the Earth surface at the Earth’s pole). Nowadays, the observation sampling being much finer, it is
possible to estimate diurnal, and even sub-diurnal, Earth rotation and orientation variations. Instantaneous observation of the Earth rotation and orientation and their history would provide us with the position of the instantaneous rotation axis in space as well as in a terrestrial frame (it is then called wobble). Those measurements, in the ITRF, contain both the sub-diurnal polar motion (due to rapid motion in the ocean-atmosphere, for instance) and low frequency nutations in the ICRF which appear diurnal in the ITRF due to the convolution with the Earth rotation. Conversely, the sub-diurnal motions, in the ICRF, include both the sub-diurnal nutation (as created for instance by the irregular shape of the Earth) and low frequency polar motion in the ITRF which, expressed in the ICRF, appears nearly diurnal. To separate properly polar motion and nutation, they had to be redefined: the nutation, as previously, includes all the motion with frequency corresponding to long periods in the celestial frame, i.e. to diurnal retrograde periods in the terrestrial frame. Polar motion is now defined as motions in all the frequency bands in the ITRF, except the interval [-0.5, -1.5] cycles per day (IERS Conventions 2003). In view of these definitions, we now explain how it is possible to pass from one reference frame to the other using precession/nutation, Earth rotation speed or Earth rotation angle, and polar motion.

Let us define some fundamental planes and axes: first, the instantaneous rotation axis is the axis of points that are not affected by the Earth rotation at any given time. Obviously, this axis moves with time with respect to the terrestrial frame as the Earth wobbles around its rotation axis; this motion is called the (polar) wobble. Then, we define an axis whose motion is the same as the rotation axis, but smoothed over a one-day period and consistent with the definition of polar motion and nutation explained in the previous paragraph. This axis is called the Celestial Intermediate Pole (CIP) axis in the recently adopted definitions (IAU and IUGG resolutions) and is a conceptual axis of fundamental importance. In addition, the z-axis of the terrestrial reference frame is oriented along the principal moment of inertia of the Earth or figure axis, and its counterpart in the celestial frame is the normal to the equator at the reference epoch, corrected for the nutation terms (so-called mean equator at J2000). The transformation can, in principle, be done in three successive steps, for both the classical and the new methods.

(1) By a transformation bringing the pole of the equatorial frame of the terrestrial frame on the intermediate pole, corresponding to the motion of the intermediate pole (CIP) inside the terrestrial frame, we position the z-axis on the axis corresponding to this intermediate pole (3D-animation called http://www.astro.oma.be/D1/DIDAC/NROtext/zTRF-CIPtransformation.wmv

(2) By a rotation around this new z-axis (oriented in the direction of the intermediate pole), we account for the Earth rotation (either for the Earth Rotation Angle or for the Greenwich Sidereal Time).

(3) By a transformation bringing the new equatorial plane on the celestial equator, we correct for the motion of the intermediate pole in space, we position the intermediate pole on the Z-axis of the celestial frame (see 3D-animation called http://www.astro.oma.be/D1/DIDAC/NROtext/ZCRF-CIPtransformation.wmv

It is not a direct transformation from one reference frame to the other and two intermediate reference frames are needed. One intermediate reference frame is rotating with the Earth and the other one is non-rotating; they share the same xy-plane, but they
differ as their x-axes are separated by the Greenwich Sidereal Time (GST) or the Earth Rotation Angle (ERA). The three transformations needed can be chosen differently according to the choice of the intermediate pole. The new and classical approaches for the expressions of these three rotations consider different definitions for the intermediate frames.

2.1. The classical transformation
According to the classical transformation, the intermediate pole is the CIP. The first step is quite obvious: using the polar motion, we rotate around an equatorial axis to position the z-axis on the CIP. We make the next rotation around the CIP to account for the Earth rotation, of an angle corresponding to the GST, which brings the x-axis to the true equinox of date, i.e. the intersection between the true equator of date and the true ecliptic of date. Then, correcting for the nutation in longitude and obliquity, we move along the mean equator of date and bring the x-axis on the mean equinox of the date. Correcting for precession and obliquity rate, we eventually reach the mean equator of the reference epoch and the mean equinox of the reference epoch, which has been chosen at noon on January 1st, 2000 (called J2000). The transformation is shown in the 3D-animation called [http://www.astro.oma.be/D1/DIDAC/NROtext/ClassicalTransformation.wmv](http://www.astro.oma.be/D1/DIDAC/NROtext/ClassicalTransformation.wmv).

The complexity of this procedure lies in the fact that changing from a reference frame based one equinox to another reference frame based on another equinox is not an easy task, as we cannot do it by only one rotation around one of the axes of the reference frame. Of course, there are many possibilities. One possibility that was used before the implementation of the IAU2000 resolutions is shown on the movie and another much simpler one that limits the number of steps is represented in Figure 1. Let us start with the equator 1, the true equator of date, plane perpendicular to the z-axis in the starting reference frame, and with the ecliptic 1, plane of the Earth orbit of date (true ecliptic of date). The equinox 1, \( \gamma_1 \) (=\( \gamma_D \) in the literature), is the intersection between those two planes. We want to eventually reach a reference frame 2, defined by the intersection \( \gamma_2 \) (=\( \gamma_0 \) in the literature) between the equator 2 and the ecliptic 2, both corresponding to the mean planes at another epoch of reference.

1) We define an intermediary equinox, at the intersection \( \gamma_{int} \) between equator 1 (of date) and ecliptic 2 (of epoch). The reference frame is then rotated around the z-axis in order to move the x-axis on \( \gamma_{int} \). The y-axis is moved accordingly.

2) Rotating around the \( \gamma_{int} \) axis, we rotate around the x-axis so that the y-axis belongs to the ecliptic 2 plane, which thus becomes the new xy plane.

3) By a z-rotation around the perpendicular to the ecliptic 2 frame, we move \( \gamma_{int} \) along the ecliptic 2 to bring it on the equinox 2, \( \gamma_2 \) (=\( \gamma_0 \)).

4) Rotating again around the new x-axis (i.e. around \( \gamma_0 \)), we move the new y-axis in the equator 2 plane, which ends the transformation.

As we have to do it twice (once to go from the true equinox of the date \( \gamma_1 = \gamma_D \) to the mean equinox of the date \( \gamma_{int} \), and once to go from that equinox to the equinox at J2000, \( \gamma_2 = \gamma_0 \)), it makes the transformation uneasy, which constitutes a considerable drawback. Another disadvantage of this method is the use of equinox, as (1) a precise definition of the ecliptic is a difficult issue, (2) the motion of the equinox, being the intersection of two
moving planes, is not an easy one, and (3) the rotation of the Earth provided by the GST is different from one epoch to the other even if one does not have real length-of-day variation, i.e. the equinox motion is contaminated by precession and nutation. All the steps needed for this transformation are shown in the movie 1.

Figure 1: Four successive rotations are needed to change from the true equinox of date to the mean equinox of J200.0.

2.2. Transformation based on the non rotating origins

The new transformation is based on the same intermediate pole but on another intermediate frame, with other definitions of the $x$-axis of both the celestial frame (called the celestial intermediate origin, CIO, Non Rotating Origin with respect to the CRS) and the terrestrial frame (called the terrestrial intermediate origin, TIO, Non Rotating Origin with respect to the TRS).

These origins are points of the CIP equator chosen so that they have no motion along the CIP equator associated with polar motion and precession/nutation. This condition imposes a relation between the polar motion and the motion of the TIO at the Earth surface, and a relation between the precession/nutation and the motion of the CIO in space, so that the effects of the polar motion and precession/nutation on the Earth rotation rate are corrected for. This transformation is shown in the 3D-animation called http://www.astro.oma.be/D1/DIDAC/NROtext/NewTransformation.wmv This choice does not impose any constraint on the point chosen on the CIP equator, but only on its
motion: when the Earth axis is moving either in the celestial frame or in the terrestrial frame, the non-rotating origin itself has to mirror this motion in space or at the Earth surface, so that it does not move along the equator of the CIP. There is consequently no effect from the precession and nutation on the CIO motion. Similarly, from the motion of the TIO is not affected by the polar motion. The Earth rotation rate is then defined as the time variation of the angle between the non-rotating origins, between the CIO and the TIO, this angle is called the Earth Rotation Angle (ERA). As the motion of the NRO is imposed by the no-rotation condition, none of the points defined geometrically can be used as such. The 3D-animation called http://www.astro.oma.be/D1/DIDAC/NROtext/AllMotionNRO.wmv shows, in the case of precession/nutation of the CIO, that they do have a motion along the CIP equator. Let us discuss the motion of those points, which are mainly defined by geometrical conditions, except for the CIO, defined by a kinematical condition. They are shown in Figure 2. $\Sigma_0$ is the intersection of the reference meridian (close to the mean equinox of J2000) and the equator of the ICRF (mean equator of epoch J2000). Obviously, this point cannot be chosen, as it does not belong to the CIP equator. The intersection of the CIP equator with the ICRF reference meridian, $K$, moves along the CIP equator to stay on the reference meridian, which disqualifies it. $H$ is the intersection of the CIP equator with the circle passing through the CIP and $\Sigma_0$ and, similarly to $K$, moves along the CIP equator to follow its definition. Similarly, $\Sigma$, the point at the same distance of the node (intersection between the ICRF equator and the CIP equator) as $\Sigma_0$ cannot be chosen, as it moves along the equator. Those points, which could be a good choice for being a NRO at the reference epoch, cannot be used as a NRO, as they do not have the right motion. Consequently, we have to choose another point, which is defined to have the right motion. We are free, at the basic epoch, to choose the point arbitrarily. We use this degree of freedom to ensure the continuity of the definition of the origin of the intermediate frame. We define the motion of the non-rotating origins with respect to the celestial and terrestrial reference frame as the quantity $s$, and $s'$, respectively.
2.3. The new way of changing reference frames
The new method for changing reference frames (shown in the 3D-animation called http://www.astro.oma.be/D1/DIDAC/NROtext/NewTransformation.wmv as already mentioned), as it does not refer to the equinox anymore, is much easier. As a first step, the CIP motion in the Earth reference frame is accounted for in order to move $z$ to the CIP (rotation around the $x$-axis). Then, the rotation $s'$ around the same $z$-axis is applied in order for $x$ to be brought on the non-rotating origin $\varpi$ (corresponding to the TIO). The third step consists in a rotation around the CIP axis of the so-called Earth Rotation Angle (ERA, written $\theta$), in order to move $\varpi$ on the celestial counterpart of the non-rotating origin, $\sigma$ (corresponding to the CIO). The rotation $s$ is then applied, still around the CIP, and finally, the celestial motion of the CIP is taken into account in order to reach the celestial reference frame (rotation around the CIO $\sigma$ to move the CIP and on the Z-axis of the ICRF). Inside the motions $s$ and $s'$ are hidden contributions associated with polar motion and nutation respectively and are easily solved for kinematically (see Capitaine et al., 2003) from the polar motion and the nutation. The reason for the simplicity of the new approach lies into two facts: (1) to go from the ITRF to the intermediate frame involving the non rotating origin $\omega$, one can use a rotation using polar motion components only, and (2) to go from the non rotating origin $\sigma$ to the ICRF, one can use a rotation using nutation and precession components only. Figure 3 shows the different transformations involving the NROs.
2.4. Usefulness of the NRO
There are two main advantages to use Non Rotating Origins (NROs). First, we do not need to consider the equinox as a basis for the transformation, as explain in Section 2.1, which is good as the motions of the ecliptic and of the equators are mapped into the equinox. The definition of the equinox is ambiguous, and its evolution with time makes troubles. Second, and this is the main reason for having established it, the monitoring of the Earth rotation rate variations should not incorporate other effects such as effects related to variations in orientation. The NRO definition allows a total decoupling of the Earth Rotation Angle (ERA) variation from the precession-nutation and polar motion. In the previous approach, the GST was a mixture between the rotation rate variation and projections of the rotation axis motion, i.e. the precession and nutation of the equinox or the polar motion. The previous contaminations of precession-nutation and of polar motion have been absorbed in the motion $s'$ and $s$. The main advantage of this formalism is thus that the ERA corresponds to the actual Earth rotation, without any perturbation from polar motion or precession-nutation, which was the case with GST when the equinox was chosen for the intermediate frames.
3. Additional refinement

The ICRF is defined using VLBI source coordinates, so that it was the closest to the mean equator of J2000 at the time of its realization. This does not mean that the present realization of the ICRS will be the same as when it was adopted, due to additional accumulation of data. Actually offsets have been determined for the ICRF CIP coordinates at J2000. This constant offset is of the order of 20 milliarcseconds (see IERS Conventions 2003).

It must be noted that in our graphic/movie 3D representation, the sub-diurnal motions are neglected. Considering these motions highly complicate everything, but the ideas remain very similar.

4. Conclusions

To study the sky from Earth, we need two reference frames, a celestial reference frame, as fixed as possible with respect to the inertial frame, and a terrestrial reference frame, rotating with the Earth, and we need a way to transform coordinates from one reference frame to the other. This transformation involves the Earth rotation rate, the polar motion, and the precession-nutation. This transformation is done using intermediate frames differing from each other by the Earth rotation rate. The more obvious transformation would not mix the components Earth Rotation Angle (ERA), polar motion, and precession-nutation. To that aim, the concept of non rotating origin (NRO) has been introduced. The use or not of NROs affects the intermediate frame only; the ICRF and ITRF definition are not changed. There are a lot of NRO candidates, as the property of being non-rotating only imposes a given motion of the origins - the motion of the Celestial Intermediate Origin (CIO) along the equator must remain unaffected by precession and nutation; the motion of the Terrestrial Intermediate Origin (TIO) along the equator must remain unaffected by polar motion – but not the points themselves at the reference epoch. For practical observation purpose, the NROs have been defined by choosing an initial point so that continuity with earlier convention can be achieved on January 1st, 2003 (date of the imposed change by the IAU resolution). The new transformation system is much simpler, not because it refers to the NRO, but because it does not imply the use of the equinox. Nevertheless, the use of the NROs allows an elegant separation between the polar motion, the precession nutation and the rotation rate variation. Or in other words, the enormous advantage of this new approach is that the Earth rotation rate can be directly deduced from the ERA. There is presently an international effort for clarification of the terminology associated with the implementation of the IAU2000 resolution (Capitaine et al., 2006).

Bibliography


